Cardinality

Monday, August 31, 2020 9:32 AM

Cardinality of Sets

Cardinality of a set S, denoted by |S|, is the number of distinct elements of the set. The number is also referred as the cardinal number. If a set has an infinite number of elements, its cardinality is ∞. The cardinality of the set *A* is often notated as |*A*| or n(*A*)

Example: | {1, 4, 3, 5}| = 4, |{1, 2, 3, 4, 5,…}| = ∞

Power set of a set:

Let A be any set. The power set of A is the set P(A) = {x : x ⊂ A}. In words, the power set of A is the set of all the subsets of A. (also including null and the original set with the subsets)

Example: If A = {1,2}, then P(A) = { ∅, {1}, {2}, { 1, 2}}

Note:

- For any set A we have ∅ ∈ P(A) and A ∈ P(A), so P(A) is non empty for every set A.
- Power set of a finite set is finite.
- If a set has 'n' elements then the number of Power Sets present for the given set is 2^n

Partition of Sets

Let S be a nonempty set. A partition of S is a subdivision of S into non overlapping, nonempty subsets. Precisely, a partition of S is a collection { Ai } of non-empty subsets of S such that:

(i) Each element in S belongs to one of the Ai.

(ii) The sets of { Ai } are mutually disjoint; that is, if

The subsets in a partition are called cells.

(Maximum number of partition cell is equal to the number of elements in the set)

$$A_j \neq A_j \text{ then } A_j \cap A_k = \Phi$$

For example: Find the partition of A = { 1, 2}

Partition containing one cell = { 1, 2}

Partition containing two cells = { {1}, {2} }

Countable set

A set is called countable when its element can be counted. A countable set can be finite or infinite.

Power set of countably finite set is finite and hence countable.

For example S = { a, e, i, o, u}

Set S representing vowels has 5 elements and its power set contains 2^5 = 32 elements. Therefore, it is finite and hence countable.

Power set of countably infinite set is uncountable.

For example S = { 1, 2, 3, 4, ….. ∞}

Set S representing set of natural numbers is countably infinite. However, its power set is uncountable.

Uncountable set

A set is called uncountable when its element can't be counted. An uncountable set can be always infinite.

For example Set S containing all fractional numbers between 1 and 10 is uncountable.

Power set of uncountable set is always uncountable.

(minimum means intersection)
(maximum means union)

Minimum Set or Minset or Minterm

Let A be any non - empty set and { B1, B2,……..,Bn } be any subsets of A. Then the minimum set generated by the collection { B1, B2,……,Bn } is a set of the type D1 ∩ D2 ∩,……..,Dn where each D1, D2,…..,Dn is Bi or Bic for i = 1, 2, 3,_ _ _ _,n).

Remark

1) The number of Minsets generated by n sets is 2^n. (n is the number of subsets)

2) The collection of all the nonempty Minsets generated y given subsets gives rise the partitions of the set.

For better understanding: Let S be any non - empty set { 1, 2, 3, 4 } and B1 and B2 be its subsets.
Then Minset is
D1 = B1 ⋂ B2
D2 = B1' ⋂ B2
D3 = B1 ⋂ B2'
D4 = B1' ⋂ B2'

Maximum Set or Maxset or Maxterm
Let S be any non - empty set and { B1, B2,……..,Bn } be any subsets of S. Then the maximum set generated by the collection { B1, B2,……,Bn } is a set of the type A1 U A2 U,……..,An where each A1, A2,…..,An is Bi or Bic for i = 1, 2, 3,_ _ _ _,n).
Remark
1) The number of Maxsets generated by n sets is 2^n. (n is the number of subsets)
2) The collection of all the nonempty Maxsets generated y given subsets gives rise the partitions of the set.

For better understanding: Let S be any non - empty set { 1, 2, 3, 4 } and B1 and B2 be its subsets.
Then Minset is
A1 = B1 U B2
A2 = B1' U B2
A3 = B1 U B2'
A4 = B1' U B2'

Ordered Pair
An ordered pair (*a*, *b*) is a pair of objects. The order in which the objects appear in the pair is significant: the ordered pair (*a*, *b*) is different from the ordered pair (*b*, *a*) unless *a* = *b*.

Cartesian Product / Cross Product of two sets
René descartes invented Cartesian Product. He formulated analytic geometry which helped in the origination of the concept. The Cartesian product of two sets A and B, denoted A × B, is the set of all ordered pairs (a, b) where a is in A and b is in B.
In terms of set-builder notation, that is
A x B = { (a, b): a ∈ A and b ∈ B}

Note:
- The Cartesian product A × B is not commutative,
- The Cartesian product is not associative (unless one the set is empty)
- The Cartesian product is not commutative.
- |A × B| = |A| · |B|

Principle of Inclusion and Exclusion
The principle of inclusion and exclusion computes the cardinal number of the union of the multiple non disjoint sets.
- For two sets A and B, the principal states |A U B| = |A| + |B| - |A ∩ B|
- For three sets A, B and C, the principal states |A U B U C| = |A| + |B| - |A ∩ B| - |B ∩ C| - |A ∩ C| + |A ∩ B ∩ C|

PROOF
We know that A U B is the union of three disjoint sets, A-B, B-A AND A ∩ B
Therefore, |A U B| = |A-B| + |B-A| + |A ∩ B| ……(i)
Again, A is the union of A-B and A ∩ B, which are disjoint sets
Therefore, |A| = |A-B| + |A ∩ B| ………(ii)
Similarly, |B| = |B-A| + |A ∩ B| ……..(iii)

Adding (ii) and (iii)

|A| + |B| = |A-B| + |B-A| +2|A ∩ B|
= (|A-B| + |B-A| + |A ∩ B|) + |A ∩ B|)
= |A U B| + |A ∩ B|
Thus, |A U B| = |A| + |B| - |A ∩ B|

- For two disjoint sets A and B, the principal states, |A U B| = |A| + |B|
- For three disjoint sets A, B and C, the principal states, |A U B U C| = |A| + |B| + |C| - |A ∩ B|- |B ∩ C|+ |A ∩ B ∩ C|